

Answer the following questions. Calculators and mobile telephones are not allowed.

1. The region R is bounded by the following curves: $y = \tan x$, $y = 1$ and $x = 0$. Sketch the region R and find the volume of the solid generated by revolving the region R about the line $y = 1$. (5 points)

2. Differentiate the following function $f(x) = (\cos^{-1} y + 1)^x + x^{\operatorname{csch}(y)}$ (4 points)

3. Let $f(x) = \frac{2^x - 2^{-x}}{2} + 2$. Show that f^{-1} exists. Find $f^{-1}(x)$, state its domain and its range. (4 points)

4. Show that the only functions $f(x)$ such that $f'(x) = f(x)$ are the functions Ce^x , where C is a constant. (4 points)

5. Answer ONLY ONE PART (either (a) or (b)). (5 points each)

(a) Find the area of the region that is inside the graph of $r = \sin \theta$ and outside the graph of $r = 1 - \sin \theta$.

(b) Find the length of the curve $x = \sin t$, $y = \frac{1}{4} \cos 2t$, where $0 \leq t \leq \frac{\pi}{2}$.

6. Determine whether the integral $\int_0^{\infty} e^{-x} \sin x dx$ converges or diverges, and if it converges find its value. (4 points)

7. Discuss and sketch the graph of the equation (4 points)

$$x^2 - 4y^2 - 4x - 8y - 4 = 0$$

8. Evaluate $\lim_{x \rightarrow \infty} \left[\frac{(3)^{\frac{1}{x}} + (12)^{\frac{1}{x}}}{2} \right]$ (4 points)

9. Evaluate the following integrals: (4 points each)

(a) $\int \frac{1}{2 + 2 \cos x + \sin x} dx$

(b) $\int \frac{\ln(\tan^{-1} \sqrt{x})}{x^{\frac{1}{2}} + x^{\frac{3}{2}}} dx$

(c) $\int \frac{x^4 + 1}{x(x^2 + 2x + 2)} dx$

(d) $\int \frac{\sec x}{\cos x} dx$